A New Radio Frequency Angular Tropospheric Refraction Model

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In the previous Deep Space Network Progress Report a new angular tropospheric refraction model that very accurately reflected precise optical refraction data was presented. In this report, the above optical refraction model is transformed to a refraction model applicable at radio (S- and X-Band) frequencies. The accuracies of this new model are:

$$
\begin{aligned}
1\sigma \textit{ uncertainty} &\approx 0.002\, deg & EL &\geq 5\, deg \\
&\approx 0.005\, deg & 5 > EL &\geq 0\, deg \\
&\approx 0.015\, deg & 0 > EL &\geq -3\, deg
\end{aligned}
$$

I. Introduction

In a previous article (Ref. 1) the authors presented a new angular tropospheric refraction model which very accurately modeled existing optical angular refraction data, as follows:

$$R = F_p F_t \left(\exp \left\{ \frac{\sum_{j=0}^{8} K_{j+3} \left[U(Z) \right]^j}{1 + \Delta_3(Z)} \right\} - K_{12} \right)$$

$$F_p = \left(\frac{P}{P_0} \left\{ 1 - \frac{\Delta_1(P,Z)}{1 + \Delta_3(Z)} \right\} \right)$$

$$F_t = \left(\frac{T_0}{T} \left\{ 1 - \frac{\Delta_2(T,Z)}{1 + \Delta_3(Z)} \right\} \right)$$

$$\Delta_1(P,Z) = (P - P_0)\{\exp [A_1(Z - A_2)]\}$$

$$\Delta_2(T,Z) = (T - T_0)\{\exp [B_1(Z - B_2)]\}$$

$$\Delta_3(Z) = (Z - C_0)\{\exp [C_1(Z - C_2)]\}$$

where

R = refraction, sec

Z = actual zenith angle, deg

EL = elevation angle

EL = 90 deg − Z

$$U(Z) = \left\{\frac{Z - K_1}{K_2}\right\}$$

$K_1 = 46.625$

$K_2 = 45.375$

$K_3 = 4.1572$

$K_4 = 1.4468$

$K_5 = 0.25391$

$K_6 = 2.2716$

$K_7 = 1.3465$

$K_8 = -4.3877$

$K_9 = 3.1484$

$K_{10} = 4.5201$

$K_{11} = -1.8982$

$K_{12} = 0.89000$

P = pressure, mm Hg

P_0 = 760.00 mm Hg

$A_1 = 0.40816$

$A_2 = 112.30$

T = temperature, kelvins

T_0 = 273.00 K

$B_1 = 0.12820$

$B_2 = 142.88$

$C_0 = 91.870$

$C_1 = 0.80000$

$C_2 = 99.344$

More pertinent for use at JPL, however, would be an angular refraction model which would possess very high accuracy at S- and X-band (radio) frequencies. At the time, it was hoped that a reasonably accurate method could be found to transform the optical refraction model to a radio-frequency refraction model. Past attempts to accomplish this will be dealt with first, and then a new method to accomplish the transformation will be proposed.

II. Past Attempts to Transform Angular Refraction Models From Optical to Radio Frequencies

To facilitate a discussion of past attempts to generate angular tropospheric refraction models for use at radio frequencies, let the following notation be introduced:

$R_{OP} = R(P,T,Z)$ = optical refraction model from above

$R_{RF} = R_{RF}(P,T,Z,RH)$ = radio frequency refraction model

P = pressure

T = temperature

Z = zenith angle

RH = relative humidity

$N(h) = ND(h) + NW(h)$

$N(h)$ = total refractivity at radio frequencies

$ND(h)$ = dry, or optical component, of refractivity

$NW(h)$ = wet component of refractivity

h = height

h_0 = station height

s = parameter surface value

$ND(h_0) = ND_s$

$NW(h_0) = NW_s$

$N(h_0) = N_s$

In general, attempts to construct a radio frequency refraction model consisted of appropriating an empirical model from optical refraction work which would give the functional dependence on Z (say $R_Z(Z)$), and then scaling this expression by the total radio frequency surface refractivity, i.e.,

$$\text{refraction} \approx \left(\frac{N_s}{N_R}\right) R_Z(Z)$$

where N_R = reference optical refractivity.

At this point, one must ask, what are the implications of this procedure? Since any signal (that is of interest here) must traverse the entire troposphere, and is of course, continually being refracted, one might think that instead of being proportional to surface refractivity, angular refraction is really more nearly proportional to total (integrated) tropospheric refractivity, i.e.,

$$\text{Refraction} \propto \int N(h)\, dh$$

However, refraction could also be proportional to surface refractivity if it could be assumed that there exists some $f(h)$ such that:

$$N(h) \approx N_s f(h)$$

so that

$$\text{refraction} \propto \int N_s f(h)\, dh = N_s \int f(h)\, dh$$

Making the assumption that

$$ND(h) \sim ND_s f_1(h)$$

$$NW(h) \sim NW_s f_2(h)$$

one would have for the optical case:

$$\begin{aligned} R_Z \propto \int ND(h)\, dh &= \int ND_s f_1(h)\, dh \\ &= ND_s \int f_1(h)\, dh \end{aligned}$$

For the radio frequency case:

$$\begin{aligned} R_{RF} \propto \int N(h)\, dh &= \int \{ND(h) + NW(h)\}\, dh \\ &= \int \{ND_s f_1(h) + NW_s f_2(h)\}\, dh \\ &= ND_s \int f_1(h)\, dh + NW_s \int f_2(h)\, dh \end{aligned}$$

Without precise knowledge of the form of $f_1(h)$ and $f_2(h)$, the only way that the surface refractivities could be used to transform from the optical case to the radio case would be if

$$f_1(h) \approx f_2(h)$$

Then

$$R_{RF} \propto (ND_s + NW_s) \int f_1(h)\, dh$$

and indeed

$$R_{RF} \propto (ND_s + NW_s) \left[\frac{R_Z(Z)}{ND_s} \right]$$

However, it is well known that wet refractivity "decays" much more rapidly than dry refractivity (for instance, Ref. 4), so that $f_1(h)$ and $f_2(h)$ are quite dissimilar; thus, the procedure of scaling an optical angular refraction model by the total surface radio refractivity to achieve a radio angular refraction model would appear to be seriously flawed.

III. Method Used to Transform From an Optical to a Radio Frequency Refraction Model

From the previous section it was seen that

$$R_{RF} \propto N_s = ND_s + NW_s$$

is a poor choice. A more logical choice would be

$$\begin{aligned} R_{RF} \propto \int N(h)\, dh &= \int \{ND(h) + NW(h)\}\, dh \\ &= \int ND(h)\, dh + \int NW(h)\, dh \\ &= \int ND(h)\, dh \left\{ 1 + \frac{\int NW(h)\, dh}{\int ND(h)\, dh} \right\} \end{aligned}$$

Similarly, for the optical case (using the model previously presented):

$$R_{OP} \propto \int ND(h)\, dh$$

Combining the above, one arrives at the equation that will be used for the radio frequency angular refraction model:

$$R_{RF}(P,T,Z,RH) \approx R_{OP}(P,T,Z) \left\{ 1 + \frac{\int NW(h)\, dh}{\int ND(h)\, dh} \right\}$$

IV. Determination of Ratio of Integrated Wet Refractivity to Integrated Dry Refractivity

In attempting to determine an analytical parametric representation for the expression:

$$\frac{\int NW(h) dh}{\int ND(h)\, dh}$$

The most difficult problem by far lies with the integrated wet refractivity. Berman first showed in 1970 (Ref. 2) that

$$\int ND(h)\, dh = AP_s \left[\frac{R}{g} \right]$$

where

$A = 77.6$

P_s = surface pressure, mbar

R = perfect gas constant

g = gravitational acceleration

$g/R = 34.1\,^{\circ}\mathrm{C/km}$

and also gave an expression to approximate the integrated wet refractivity:

$$\int NW(h)\,dh =$$

$$\left[\frac{C_1 C_2 (RH)_s}{\gamma}\right]\left\{\frac{\left(1-\frac{C}{T_0}\right)^2}{B-AC}\right\}\exp\left(\frac{AT_0-B}{T_0-C}\right)$$

where

$C_1 = 77.6$

$C_2 = 29341.0$

RH = relative humidity

γ = temperature lapse rate

$C = 38.45$

T_0 = extrapolated surface temperature

$A = 7.4475 \ln(10)$

$B = 2034.28 \ln(10)$

Chao (Ref. 5) later improved upon the integrated wet refractivity with the expression:

$$\int NW(h)\,dh = 1.63 \times 10^2 \left\{\frac{e_0^{1.23}}{T_0^2}\right\} + 2.05 \times 10^2\,\alpha \left\{\frac{e_0^{1.46}}{T_0^3}\right\}$$

where

e_0 = surface vapor pressure, $\mathrm{N/m^2}$

T_0 = surface temperature, K

α = temperature lapse rate, K/km

However, both of these expressions depend upon one or more parameters not measurable at the surface (i.e., temperature lapse rate, etc.), and neither is particularly accurate. Going back to the previous section, if the altitude-dependent refractivities could really be represented as

$$ND(h) \sim ND_s f_1(h)$$

$$NW(h) \sim NW_s f_2(h)$$

and if the above refractivities could be integrated, i.e., if A below could be evaluated as

$$A = \frac{\int f_2(h)\,dh}{\int f_1(h)\,dh}$$

then one might simply expect that

$$\frac{\int NW(h)\,dh}{\int ND(h)\,dh} \approx A\left\{\frac{NW_s}{ND_s}\right\}$$

To test this hypothesis, the authors had ten cases used in Ref. 2. Although a very small number, the cases were alternate day and night profiles selected throughout the year (December, February, April, August, September). A least-squares linear curve fit to the above data was performed as follows:

$$\frac{\int NW(h)\,dh}{\int ND(h)\,dh}; \qquad A\left\{\frac{NW_s}{ND_s}\right\}$$

The fit yielded the following:

$A = 0.3224$

$\sigma(\%) = 00.93\%$

$$\times\left[\sigma(\%) = 100 \times \sigma\left(\frac{\int NW(h)\,dh}{\int ND(h)\,dh} - A\left\{\frac{NW_s}{ND_s}\right\}\right)\right]$$

Translated to centimeters of integrated refractivity, one would have

$$\sigma(\mathrm{cm}) = 2.0\ \mathrm{cm}$$

Table 1 and Fig. 1 present the detailed analysis of the ten cases described.

As a totally independent check of this observed relationship, use can be made of work done by Chao (Ref. 4) on wet and dry refractivity profiles. Combining Eqs. (9), (10), (13), (14), (15), and (16) from Ref. 4, one has:

$$ND(h) = ND_s\left(1 - \frac{h}{42.7}\right)^4 \qquad h \le 12.2\ \mathrm{km}$$

$$= \frac{70}{269} ND_s \left\{\exp\left(-\frac{(h-12.2)}{6.4}\right)\right\} \qquad h \ge 12.2\ \mathrm{km}$$

$$NW(h) = NW_s\left(1 - \frac{h}{13}\right)^4 \qquad h \le 13\ \mathrm{km}$$

$$= 0 \qquad h \ge 13\ \mathrm{km}$$

Performing the dry refractivity integration, one has

$$\int_0^\infty ND(h)\,dh = \int_0^{12.2} ND_s \left(1 - \frac{h}{42.7}\right)^4 dh$$

$$+ \int_{12.2}^\infty \frac{70}{269} ND_s \left\{\exp\left(-\frac{(h-12.2)}{6.4}\right)\right\} dh$$

$$= ND_s \int_0^{12.2} \left(1 - \frac{h}{42.7}\right)^4 dh$$

$$+ \frac{70}{269} ND_s \int_{12.2}^\infty \exp\left[-\frac{(h-12.2)}{6.4}\right] dh$$

transforming the first integral by

$$\left(1 - \frac{h}{42.7}\right) = x$$

$$dh = -42.7\,dx$$

so that

$$\int \left(1 - \frac{h}{42.7}\right)^4 dh = -42.7 \int x^4\,dx$$

$$= -42.7 \frac{x^5}{5}$$

$$= -\frac{42.7}{5}\left[\left(1 - \frac{h}{42.7}\right)^5\right]_0^{12.2}$$

$$= 6.952$$

Transforming the second integral by

$$-\frac{(h-12.2)}{6.4} = x$$

$$dh = -6.4\,dx$$

so that

$$\int \exp\left[-\frac{(h-12.2)}{6.4}\right] dh = -6.4 \int \exp(x)\,dx$$

$$= -6.4 \exp(x)$$

$$= -6.4\left[\exp\left(-\frac{(h-12.2)}{6.4}\right)\right]_{12.2}^\infty$$

$$= 6.4$$

Or, finally

$$\int_0^\infty ND(h)\,dh = ND_s(6.952)$$

$$+ \frac{70}{269} ND_s(6.4)$$

$$= 8.6174(ND_s)$$

Performing the wet refractivity integration, one has

$$\int_0^\infty NW(h)\,dh = \int_0^{13} NW_s \left(1 - \frac{h}{13}\right)^4 dh$$

$$= NW_s \int_0^{13} \left(1 - \frac{h}{13}\right)^4 dh$$

Transforming the integral by

$$\left(1 - \frac{h}{13}\right) = x$$

$$dh = -13\,dx$$

$$\int \left(1 - \frac{h}{13}\right)^4 dh = -13 \int x^4\,dx$$

$$= -13 \frac{x^5}{5}$$

$$= \frac{13}{5}\left[\left(1 - \frac{h}{13}\right)^5\right]_0^{13}$$

$$= 2.6$$

so that

$$\int_0^\infty NW(h)\,dh = 2.6(NW_s)$$

Combining the integrated wet refractivity and the integrated dry refractivity yields

$$\frac{\int_0^\infty NW(h)\,dh}{\int_0^\infty ND(h)\,dh} = \frac{2.6(NW_s)}{8.6174(ND_s)}$$

$$= 0.30172\left(\frac{NW_s}{ND_s}\right)$$

This is to be compared to the previously determined relationship from actual data of:

$$\frac{\int_0^\infty NW(h)\,dh}{\int_0^\infty ND(h)\,dh} \approx 0.3224\left(\frac{NW_s}{ND_s}\right)$$

Since the value of the 1σ standard deviation

$$1\sigma = 00.93\% \; (\sim 2 \text{ cm})$$

found from actual data compares favorably with the most recent modeling published by Chao in Ref. 5 ($\sim$3 cm for combined night and day profiles), and since the basic relationship seems verifiable by average profiles presented by Chao; the determined expression will be adopted for use with the optical refraction model. The surface refractivity (Ref. 2) is defined as:

$$NW_s = \frac{(RH)_s C_1 C_2}{T_s^2} \exp\left(\frac{AT_s - B}{T_s - C}\right)$$

$$ND_s = C_1 \frac{P_s}{T_s}$$

so that one would obtain

$$\left\{1 + \frac{\int_0^\infty NW(h)\,dh}{\int_0^\infty ND(h)\,dh}\right\} \cong 1 + (0.3224)\frac{(RH)_s C_2}{T_s P_s} \exp\left(\frac{AT_s - B}{T_s - C}\right)$$

To integrate this expression into the optical model, the pressure term must be converted from mbar to mm:

$$P_s(\text{mbar}) = P_s(\text{mm}) \times \frac{1013}{760}$$

so that one would finally have

$$\left\{1 + \frac{\int NW(h)\,dh}{\int ND(h)\,dh}\right\} \cong 1 + \frac{(7.1 \times 10^3)\,(RH)_s}{T_s P_s} \exp\left(\frac{AT_s - B}{T_s - C}\right)$$

where

$(RH)_s$ = surface relative humidity (100% = 1.0)

T_s = surface temperature, K

P_s = surface pressure, mm of Hg

$A = 17.149$

$B = 4684.1$

$C = 38.450$

V. Final Angular Tropospheric Radio Frequency Refraction Model

The following gives the complete radio frequency angular tropospheric refraction model:

$$R = F_p F_t F_w \left(\exp\left\{\frac{\sum_{j=0}^{8} K_{j+3}[U(Z)]^j}{1 + \Delta_3(Z)}\right\} - K_{12}\right)$$

$$F_p = \left(\frac{P}{P_0}\left\{1 - \frac{\Delta_1(P,Z)}{1 + \Delta_3(Z)}\right\}\right)$$

$$F_t = \left(\frac{T_0}{T}\left\{1 - \frac{\Delta_2(T,Z)}{1 + \Delta_3(Z)}\right\}\right)$$

$$F_w = \left(1 + \frac{W_0 RH}{TP}\left\{\exp\left[\frac{W_1 T - W_2}{T - W_3}\right]\right\}\right)$$

$$\Delta_1(P,Z) = (P - P_0)\,\{\exp\,[A_1(Z - A_2)]\}$$

$$\Delta_2(T,Z) = (T - T_0)\,\{\exp\,[B_1(Z - B_2)]\}$$

$$\Delta_3(Z) = (Z - C_0)\,\{\exp\,[C_1(Z - C_2)]\}$$

where

R = refraction, sec

Z = actual zenith angle, deg

EL = elevation angle

$EL = 90 \text{ deg} - Z$

$$U(Z) = \left\{\frac{Z - K_1}{K_2}\right\}$$

$K_1 = 46.625$

$K_2 = 45.375$

$K_3 = 4.1572$

$K_4 = 1.4468$

$K_5 = 0.25391$

$K_6 = 2.2716$

$K_7 = 1.3465$

$K_8 = -4.3877$

$K_9 = 3.1484$

$K_{10} = 4.5201$

$K_{11} = -1.8982$

$K_{12} = 0.89000$

P = pressure, mm Hg

$P_0 = 760.00$ mm Hg

$A_1 = 0.40816$

$A_2 = 112.30$

T = temperature, kelvins

$T_0 = 273.00$ K

$B_1 = 0.12820$

$B_2 = 142.88$

$C_0 = 91.870$

$C_1 = 0.80000$

$C_2 = 99.344$

RH = Relative humidity (100% = 1.0)

$W_0 = 7.1 \times 10^3$

$W_1 = 17.149$

$W_2 = 4684.1$

$W_3 = 38.450$

VI. Model Accuracies

The inaccuracies introduced by the wet refractivity term predominate over the inaccuracies presented in Ref. 1. Considering

$$1\sigma = 1.00\%$$

the maximum 1σ angular errors would be

Z, deg	ΔR, sec	ΔR, deg
0–85	6	0.002
85–90	18	0.005
90–93	50	0.015

VII. Fortran Subroutines

Reference 1 presented two Fortran subroutines, corresponding to the full optical refraction model and an abbreviated version. These two routines have been transformed to the radio frequency version of the refraction model, and are presented in Appendixes A and B. The Fortran subroutine SBEND (Appendix A) represents the full model, while XBEND (Appendix B) gives the abbreviated version. Inputs required are:

PRESS = pressure, mm of Hg

TEMP = temperature, K

HUMID = % of relative humidity (100% = 1.0)

ZNITH = actual zenith angle, deg

and the subroutines return with

R = refraction correction, sec

References

1. Berman, A. L., and Rockwell, S. T., "A New Angular Tropospheric Refraction Model" in *The Deep Space Network Progress Report 42-24*, Jet Propulsion Laboratory, Pasadena, Calif., Dec. 15, 1974.

2. Berman, A. L., "A New Tropospheric Range Refraction Model" in *Space Programs Summary*, No. 37-65, Vol. II, pp. 140–153, Jet Propulsion Laboratory, Pasadena, Calif., Sept. 30, 1970.

3. Ondrasik, V. J., and Thuleen, K. L., "Variations in the Zenith Tropospheric Range Effect Computed From Radiosonde Balloon Data" in *Space Programs Summary*, No. 37-65, Vol. II, pp. 25–35, Jet Propulsion Laboratory, Pasadena, Calif., Sept. 30, 1970.

4. Chao, C. C., "New Tropospheric Range Corrections With Seasonal Adjustment" in *The Deep Space Network Progress Report*, No. 32-1526, Vol. VI, pp. 67–73, Jet Propulsion Laboratory, Pasadena, Calif., Dec. 15, 1971.

5. Chao, C. C., "A New Method To Predict Wet Zenith Range Correction From Surface Measurements" in *The Deep Space Network Progress Report*, No. 32-1526, Vol. XIV, pp. 33–41, Jet Propulsion Laboratory, Pasadena, California, April 15, 1973.

Table 1. Surface refractivity vs integrated refractivity

$A = 0.3224 \qquad \sigma = 0.93\%$

Case	$100 \times \frac{NW_s}{ND_s}$ (%)	$A \times \left\{100 \times \frac{NW_s}{ND_s}\right\}$ (%)	$100 \times \frac{\int NW(h)\,dh}{\int ND(h)\,dh}$ (%)	Δ (%)	Δ, cm
1	4.86	1.57	2.27	+0.70	+1.48
2	3.76	1.21	2.17	+0.96	+2.03
3	5.74	1.85	1.80	−0.05	−0.11
4	5.34	1.72	1.37	−0.35	−0.74
5	4.74	1.53	1.75	+0.22	+0.47
6	7.14	2.30	2.17	−0.13	−0.28
7	24.11	7.77	8.55	+0.78	+1.65
8	31.72	10.23	9.12	−1.11	−2.35
9	7.29	2.35	4.58	+2.23	+4.72
10	9.89	3.19	2.69	−0.50	−1.06

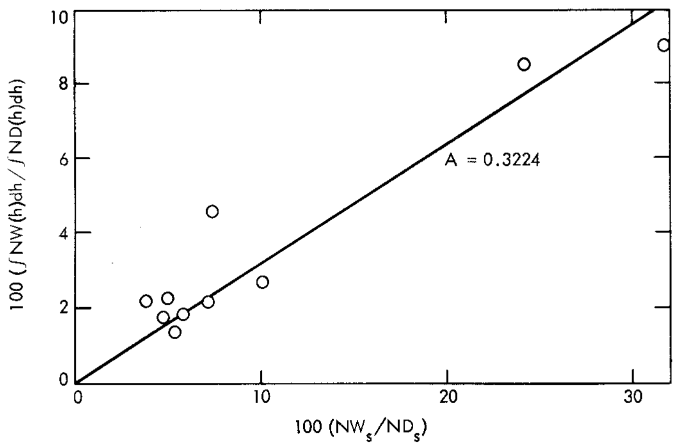

Fig. 1. Integrated refractivity vs surface refractivity

Appendix A

Subroutine SBEND

```
      SUBROUTINE SBEND(PRESS,TEMP,HUMID,ZNITH,R)
      DIMENSION A(2),B(2),C(2),E(12),P(2),T(2),Z(2)
      P(1) = 760.00
      T(1) = 273.00
      Z(1) = 91.870
      P(2) = PRESS
      T(2) = TEMP
      Z(2) = ZNITH
      A(1) = .40816
      A(2) = 112.30
      B(1) = .12820
      B(2) = 142.88
      C(1) = .80000
      C(2) = 99.344
      E(1) = 46.625
      E(2) = 45.375
      E(3) = 4.1572
      E(4) = 1.4468
      E(5) = .25391
      E(6) = 2.2716
      E(7) =-1.3465
      E(8) =-4.3877
      E(9) = 3.1484
      E(10)= 4.5201
      E(11)=-1.8982
      E(12)= .89000
      W0   = 7100.0
      W1   = 17.149
      W2   = 4684.1
      W3   = 38.450
      D3=1.+DELTA(Z,C,Z(2))
      FP=(P(2)/P(1))*(1.-DELTA(P,A,Z(2))/D3)
      FT=(T(1)/T(2))*(1.-DELTA(T,B,Z(2))/D3)
      FW=1+(W0*HUMID*EXP((W1*T(2)-W2)/(T(2)-W3))/(T(2)*P(2)))
      U=(Z(2)-E(1))/E(2)
      X=E(11)
      DO 1 I=1,8
    1 X=E(11-I)+U*X
      R=FT*FP*FW*(EXP(X/D3)-E(12))
      RETURN
      END
```

```
      FUNCTION DELTA(A,B,Z)
      DIMENSION A(2),B(2)
      DELTA=(A(2)-A(1))*EXP(B(1)*(Z-B(2)))
      RETURN
      END
```

Appendix B
Subroutine XBEND

```
      SUBROUTINE XBEND(PRESS,TEMP,HUMID,ZNITH,R)
      DIMENSION E(12)
      P     = 760.00
      T     = 273.00
      E(1)  = 46.625
      E(2)  = 45.375
      E(3)  = 4.1572
      E(4)  = 1.4468
      E(5)  = .25391
      E(6)  = 2.2716
      E(7)  =-1.3465
      E(8)  =-4.3877
      E(9)  = 3.1484
      E(10) = 4.5201
      E(11) =-1.8982
      E(12) = .89000
      W0    = 7100.0
      W1    = 17.149
      W2    = 4684.1
      W3    = 38.450
      FP=PRESS/P
      FT=T/TEMP
      FW=1+W0*HUMID*EXP((W1*TEMP-W2)/(TEMP-W3))/(TEMP*PRESS)
      U=(ZNITH-E(1))/E(2)
      X=E(11)
      DO 1 I=1,8
    1 X=E(11-I)+U*X
      R=FT*FP*FW*(EXP(X)-E(12))
      RETURN
      END
```